Int. J. Heat Mass Transfer. Vol. 21, pp. 435–443. Pergamon Press 1978. Printed in Great Britain

THE COATING OF FIBRES

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(*Received* 4 *October* 1976 *and in revised form* 22 *August* 1977)

Abstract—An experimental investigation is reported of the coating of a fibre with a liquid in which a significant part of the coating is due to freezing. Numerical methods are outlined by means of which the coated thickness is calculated by solution of the relevant Stefan's problem and the addition of viscous terms. For the system studied (biphenyl on copper) freezing accounted for between 34 and 100% of the coating, depending upon time of immersion of the fibre in the liquid and the fibre velocity. The ratio of coating thicknesses as between two different substrate radii was in good agreement with predicted values. The prediction of individual thicknesses differed from observed values by up to 20%, the agreement being poorer for higher fibre speeds.

NOMENCLATURE

a, radius of cylinder [m];
g, acceleration of gravity [m/s^2];
h_f, maximum possible thickness of deposit [m];
i, number of space intervals;
j, number of time intervals;
K, thermal conductivity [W/m °C];
L, latent heat [J/kg];
r, radius [m];
R, function of radius of curvature of meniscus, radius of frozen coating [m];
R_L, outer radius of bath [m];
t, time [s];
u, velocity of cylinder relative to liquid [m/s];
v, temperature [°C];
V_s, surface temperature of cylinder [°C];
V_0, initial temperature of cylinder [°C];
V_M, melting temperature of coating;
V_3, bath temperature at $r = R_L$;
$v_{i,j}$, temperature at point $[i\Delta r, j\Delta t]$[°C];
x, z, space parameters in Cartesians [m];
Δr, space interval [m];
Δt, time interval [s].

Greek symbols

κ, thermal diffusivity [m^2/s];
ρ, density of liquid [kg/m^3];
σ, surface tension of liquid [N/m];
η, viscosity of liquid [Ns/m^2].

1. INTRODUCTION

IN SEVERAL industrial processes a solid is coated with a liquid layer which later solidifies either by freezing, by evaporation of solvent or by chemical means. Examples of the former are the galvanizing and tin-plating of steel and certain enamelling processes, while the latter includes the application of finishes to textiles and of paints to car bodies.

The removal of a layer of viscous liquid by withdrawal of a sheet or cylinder from a bath of the former has become well understood following the work of Deryagin [1] and collaborators and of Tallmadge [2]. Recently Nicol and Wilson [3] describe modifications of the free withdrawal condition by control of the meniscus.

There are certain processes however in which factors other than the viscosity and density of the liquid and the velocity of withdrawal of the cylinder or plate are important. In these processes the liquid is near enough to its freezing point (or freezing range) for a cold substrate to cause freeze-coating, a term used by one of the authors to describe the process of coating glass fibres with molten aluminium (Arridge [4], Arridge and Heywood [5]).

In the freeze-coating process the temperature of substrate and of liquid are important parameters and the coating thickness is determined both by these and by the physical quantities (dimensions, thermal conductivity and diffusivity and latent heat) of the substrate, the coating and the liquid.

Qualitative agreement with theory was obtained for aluminium on silica and on steel and for zinc on silica. It was assumed in this theory that viscosity effects were negligible although as was pointed out by Deryagin [6], who showed experimental evidence for a thickness variation of zinc on glass rod as a function of (velocity)$^{2/3}$, the viscosity terms at high speeds are by no means negligible.

In general, of course, both processes may occur, one or the other being predominant according to the conditions of the experiment or process. For example, for low speeds near the freezing point with a cold substrate, freeze coating will probably account for the thickness of the coating, while at higher speeds, and where freezing from the substrate boundary outwards is not possible because of unsuitable temperatures, viscous boundary layer growth will be the major factor.

In the experiments to be described an attempt has been made to study the combined effect of freeze- and viscous-coating by using an organic liquid which freezes at about 70°C and measuring the thickness of coating obtained at various speeds. The theoretical

thickness was calculated numerically using the freeze coating theory together with an allowance for a viscous layer.

2. THEORY

(a) *Viscous-coating*

The theory of the deposition of a viscous liquid on to a moving substrate was developed in detail by Deryagin *et al.* in a series of papers dating from 1943.

For the case of a cylindrical rod, or fibre moving through a liquid the maximum possible thickness of deposit h_∞ is given by

$$h_\infty = \frac{2}{3}\left(\frac{\eta u}{\rho g}\right)^{1/2} \tag{1}$$

where u is the velocity of the cylinder relative to the liquid, η the viscosity of the liquid and ρ its density. Equation (1) is for the case where the capillary effect of the meniscus at the cylinder–liquid boundary may be ignored. Deryagin [7] also showed that when this capillary effect cannot be neglected the coating thickness is given by

$$h_\infty = 1.32R\left(\frac{\eta u}{\sigma}\right)^{2/3} \tag{2}$$

where R is a function of the radius of the curvature of the meniscus and σ the surface tension of the liquid. For small R the approximation $R = a$, the radius of the fibre, is valid. The condition for applicability of equation (2) is $h_\infty \ll R$, or $\eta u/\sigma \ll 1$. Verification of the 2/3 power law of dependence of coating thickness on velocity was obtained by Deryagin [6] and by Nicol and Wilson [3]. White and Tallmadge [8] also obtained the 2/3 power law dependency by a different treatment of the equations of motion.

(b) *Freeze-coating*

The build-up of a coating by freezing on to a cold substrate is known from antiquity as a method of making candles from waxes by dipping a cold wick into the molten wax and withdrawing it. When cool the coated wick is re-dipped to thicken the layer and the process continued until the required thickness of candle is obtained. We may consider the freeze-coating of a moving fibre in a similar way if we ignore the motion of the fibre, that is, if we consider the "bath" as moving with the fibre in the following way.

With reference to Fig. 1, consider an element δz of the fibre at the instant that it enters the bath. If we make the assumption (to be justified later) that heat conduction along the fibre axis is negligible in comparison with that occurring in a radial direction then the element at the instant after entering the bath is in contact with liquid at the bath temperature. In time δt a layer of thickness δr will freeze on to the surface and the element will have moved a distance $u\delta t$ vertically upwards.

If the liquid surrounding it is at the same temperature as it was lower down then we may use the radial heat conduction equations (4)–(6) below for freezing

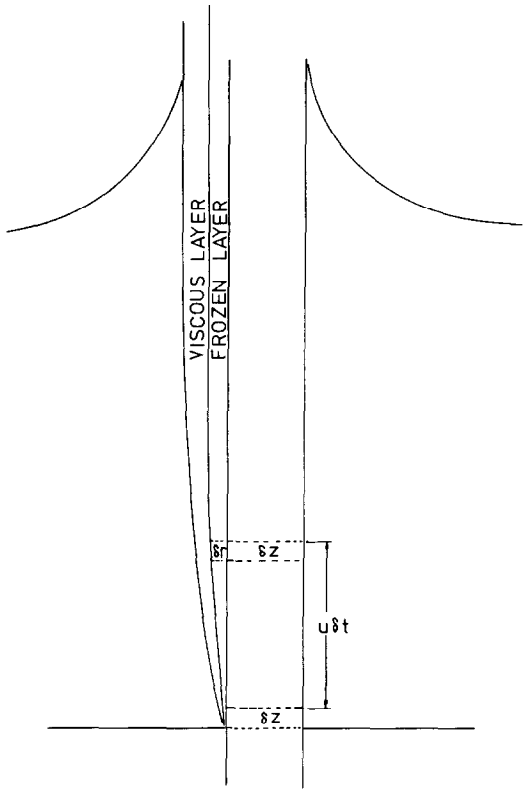

FIG. 1. Schematic diagram showing frozen-on and viscous coatings.

at the radially moving boundary, ignoring the vertical motion.

If viscosity effects play a part, as Deryagin has shown, then the moving solid boundary will have, attached to it, a viscous boundary layer which will grow in the classical manner.

Now a temperature gradient must exist in this layer, since the bath is not at freezing point. We could take this into account by considering a series of layers at different temperatures (and therefore viscosities) which will consequently modify the viscous boundary-layer thickness, increasing it from the value deduced for the viscosity of the bath.

Alternatively we could attempt to solve the equations of motion of the fluid with a viscosity that varies with the radius and a boundary that is moving.

The method we have adopted here is to calculate the thickness of the freeze coat and to add to the final value the viscous layer calculated by Deryagin's formula for the radius of the frozen layer.

The simplest way of analysing such a process is to consider the solution of an ideal heat conduction problem involving freezing at a moving boundary (a so-called Stefan problem). If we assume that conduction along the moving rod is negligible in comparison with radial conduction then we can use the equation of conduction of heat in one dimension. In Cartesian coordinates this is:

$$\frac{\partial v}{\partial t} = \kappa \frac{\partial^2 v}{\partial x^2} \tag{3}$$

where v is the temperature at point x, t the time and κ the diffusivity.

The solution of this equation when the substrate is maintained at a constant temperature is Neumann's solution of the Stefan problem (Carslaw and Jaeger [9]) and this predicts that the thickness of the frozen layer is proportional to the square root of the time.

The initial growth rate of a frozen coating should therefore follow the square root law.

In general, however, the substrate is not at constant temperature since heat will flow into the interior of the rod, raising its temperature until, eventually, the whole rod is at the temperature of the frozen layer. This will then start to melt off and the final state will be that in which the rod and melt are at a uniform temperature.

The general problem of freezing at a moving boundary, in cylindrical polar coordinates can be stated as follows.

Let region 1 be the substrate rod or fibre, region 2 the frozen-on layer and region 3 the bath of liquid. The equation of heat conduction in polar coordinates is given by

$$\frac{\partial v}{\partial t} = \kappa\left\{\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right\}. \tag{4}$$

The boundary conditions between the regions are as follows:

$$v_1 = v_2 \text{ at } r = a, \text{ for all } t \tag{5}$$

$$v_2 = v_3 = V_M \text{ at } r = R, \text{ for all } t. \tag{6}$$

Here R is the radius of the frozen-on coating and V_M melting temperature of the coating.

$$K_1\left(\frac{\partial v_1}{\partial r}\right)_a = K_2\left(\frac{\partial v_2}{\partial r}\right)_a \text{ for all } t \tag{7}$$

$$K_2\left(\frac{\partial v_2}{\partial r}\right)_R - K_3\left(\frac{\partial v_3}{\partial r}\right)_R = L\rho\frac{\mathrm{d}R}{\mathrm{d}t}, \text{ for all } t \tag{8}$$

$$v_3 = V_3 \text{ at } r = R_L, \text{ for all } t \tag{9}$$

$$\left(\frac{\partial v_3}{\partial r}\right)_{R_L} = 0, \text{ for all } t. \tag{10}$$

The initial conditions are:

$$v_1 = V_0 \text{ at } t = 0 \text{ for all } r \leqslant a \tag{11}$$

$$v_2 = V_3 \text{ at } t = 0 \text{ for all } r > a \tag{12}$$

$$v_3 = V_3 \text{ at } t = 0 \text{ for all } r > a \tag{13}$$

$$R = a \text{ at } t = 0. \tag{14}$$

These boundary and initial conditions define the problem.

The problem has no known analytic solution and numerical methods have to be used. Horvay [10], studying a similar problem, used an analogue computer method while one of the present authors Arridge and Heywood [5] has used a digital computer with a finite difference method based on that of Murray and Landis [11].

In the work presented here the Crank–Nicholson implicit method (e.g. Potter [14]) has been adopted (by H. R. L. M. Levy and A. J. Lockyer) to the problem of the moving boundary and results from it are compared with those from the older method [5].

Both iterative methods are used to calculate the overall coating thickness using first the theory of freeze-coating to calculate R and then applying the Deryagin theory with this value of R. A block schematic of the numerical calculations is given below. Computer listings are available on request.

(c) *Block schematic*

1. Define $v_{i,j}$, as the temperature at point $i\Delta r, j\Delta t$ in a space–time mesh system where the space intervals are measured radially from the centre of the rod and the time intervals taken from zero.

2. Define suitable intervals in the rod, in the frozen layer and in the liquid. Following Murray and Landis the intervals in the frozen layer increase in size with time, being an assigned fraction of $R-a$ (R is the radius of the freezing boundary and a that of the rod) while intervals in the liquid diminish in size, being related to R_L, the assumed outer radius of the liquid bath.

3. Set up difference equations as analogues to the differential equations for heat conduction and for the boundary conditions. The two methods, explicit and implicit, diverge here.

4. Determine the new value of R.

5. Determine new values of $v_{i,j}$ from the old ones.

6. Iterate 4 and 5 until total time is equal to the experimental time in bath then add viscous component.

The computational time taken in the implicit method was appreciably less than that for the explicit one.

3. EXPERIMENTAL METHOD

The choice of materials for the experiments was dictated by the necessity to compromise between materials with sharply defined melting points and low viscosity and those with higher viscosity but a wide melting range. Liquids that formed glasses rather than a crystalline phase would not freeze-coat because no latent heat would be involved.

For convenience and ease of handling biphenyl (m.p. 70°C) was chosen. This is an organic compound used as an insecticide and fungicide for citrus fruits. Its low melting point meant that temperature control was far easier than, for example, for most metals.

As substrate copper wire was chosen on the grounds of high thermal conductivity and ready availability in suitable diameters. Two different diameters were used 0.440 ± 0.004 mm and 0.365 ± 0.002 mm.

3.1. *The coating bath*

It was necessary to be able to pull lengths of wire at various speeds through a fixed height, constant temperature, bath of the biphenyl in order to test the coating theory. This was achieved by the apparatus shown schematically in Fig. 2 with details of the bath in

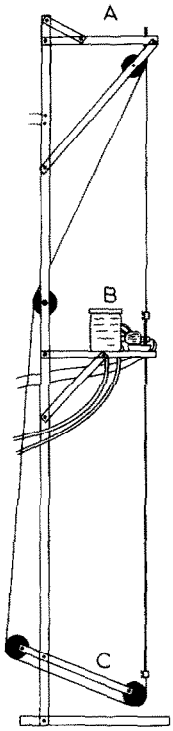

FIG. 2. Schematic diagram of coating apparatus. (A) Motor, gearing and pulleys. (B) Coating apparatus. (C) Pulley system.

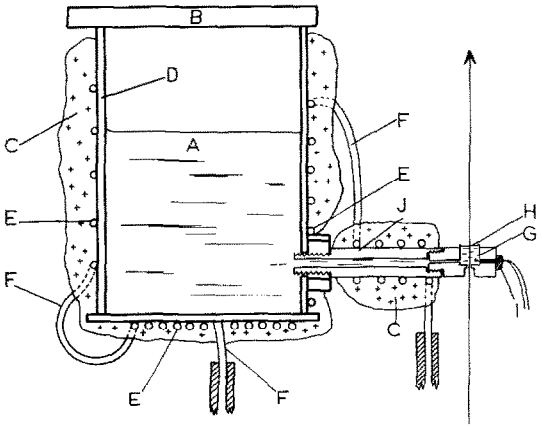

FIG. 3. Detail of coating bath and nozzle. (A) The bath of biphenyl. (B) Lid. (C) Plaster of Paris insulation. (D) Brass reservoir. (E) Copper tubing. (F) Plastic flexible tubing. (G) Brass nozzle. (H) Bath for coating. (I) Sealing wax. (J) Bath feeder tube.

Fig. 3. The bath was made entirely of brass for good thermal conductivity, covered with plaster of Paris as a heat insulator. Heating was by circulation of water, through copper tubes soldered to the bath, from a constant temperature reservoir of large capacity. The temperature of the nozzle was monitored by a thermocouple and varied no more than $\pm\frac{1}{4}$°C during each run.

In Fig. 3 the detailed design of the constant height part of the bath is shown. This consisted of a heated side tube leading out of the main receptacle and containing a nozzle which screwed into the feeder tube for ease of changing. The wire to be coated was introduced through a small hole at the bottom of the nozzle, wide enough to allow entry of the wire and to guide it but too small to allow liquid to leak out.

A constant height of bath was ensured by means of an overflow which also ensured that the bath temperature was kept steady since a constant flow of material was maintained from the larger reservoir. The diameter of the bath was about 15 times that of the wire, thereby ensuring that meniscus effects at the wall did not affect coating. (We did not aim in these experiments to study the restricted withdrawal effects studied by Nicol and Wilson.)

The wires to be coated were connected to a nylon loop passing over a pulley attached to a constant speed motor. Fast acceleration and deceleration were achieved by allowing slippage of the nylon loop during start-up or slow-down of the motor.

3.2.

The physical properties of copper and biphenyl were obtained from published tables (Kaye and Laby [12] for copper; [13] for biphenyl). The surface tension of biphenyl had to be measured however, as no published data were found. Using the capillary rise method a value of $(26.8 \pm 2.4) \times 10^{-3}$ N/m was obtained. Measurement of coating thickness was made immediately after each coating run by means of a low power microscope with a graticule eyepiece. Measurements were made at intervals along the wire, the standard deviation being about 2% of the mean. Successive experiments using identical wires, nozzles, temperatures and speeds showed reproducibility to within 5%. The variation between repeat runs being greater than that within a run.

3.3. *Experimental procedure*

The copper wire to be used was about 1.5 m in length and was washed in chromic acid and water to remove the oxide layer of unknown thermal properties before the diameter was measured. Each wire was used only once since it was found that repeated cleaning changed the diameter due to this removal of the oxide layer. The bath temperature used was 73.00 ± 0.25°C for all experiments and the copper wire was assumed to be at room temperature (21 ± 2°C) before entry to the bath. After the coating apparatus had been levelled to allow a steady overflow and sufficient time for temperature equilibrium had elapsed a coating was made and the coated fibre immediately removed for measurement.

Bath heights were changed by using different nozzles. The heights used were 3.0, 5.5, 9.0, 12.8, 18.0 and 24.5 mm.

Coating velocities used were 0.646, 0.536, 0.419, 0.333, 0.255 m/s. The values of physical properties of substrate and liquid were as follows:

Thermal diffusivity

fibre	1.128×10^{-4}	m^2/s
solid	1.714×10^{-7}	
liquid	8.064×10^{-8}	

Thermal conductivity		
fibre	3.980×10^{2}	W/m °C
solid	2.906×10^{-1}	W/m °C
liquid	1.408×10^{-1}	W/m °C
Latent heat of		
liquid	1.205×10^{5}	J/kg
Density of		
fibre	8.966×10^{3}	kg/m³
solid	1.202×10^{3}	kg/m³
liquid	1.012×10^{3}	kg/m³
Specific heat of		
fibre	3.950×10^{2}	J/kg °C
solid	1.443×10^{3}	J/kg °C
liquid	1.765×10^{3}	J/kg °C
Viscosity of		
liquid (at 71°C)	1.850×10^{-3}	Ns/m²
Surface tension of		
liquid (at 70°C)	2.68×10^{-2}	N/m
Temperature of		
fibre	21.5	°C
melting solid	70.0	°C
liquid bath	73.0	°C

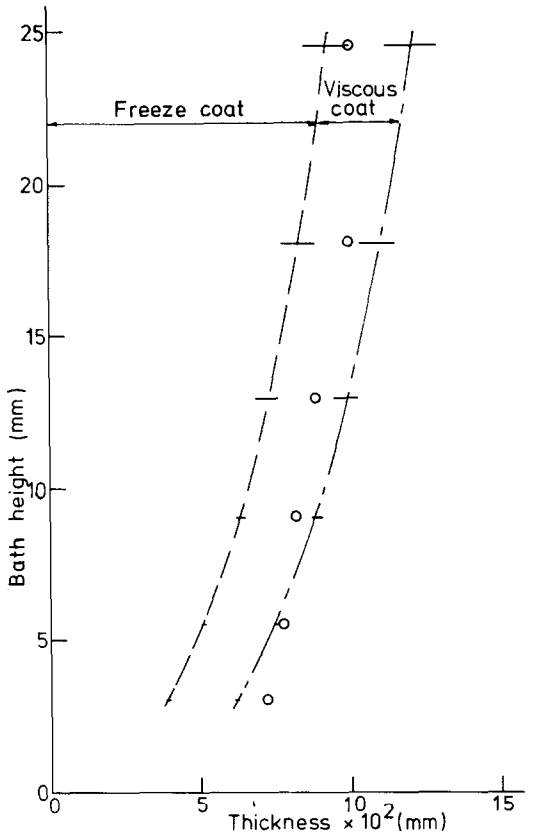

FIG. 4. Experimental and theoretical coating thickness as a function of bath height for 0.22 mm wire and velocity 0.254 m/s. Horizontal lines represent the difference between implicit and explicit methods.

4. RESULTS

Figures 4–8 show the calculated coating thicknesses for different bath heights and thread velocities for a 0.22 mm substrate. The bars shown indicate the limits of the two differently calculated values. As can be seen, at low velocities freeze coating accounts for ~90% of the total, whereas the percentage is much lower at the higher speeds. At the highest (0.646 m s) speed, globules were formed and observed thicknesses are inaccurate and are not therefore included.

Similar results to those shown in Figs. 4–8 were found for the thinner (0.183 mm) wire.

Figure 9, derived from the results for the small wire diameter, shows the proportion of the total coating due to freezing, for a range of bath heights and fibre velocities. The results for the thicker wire are similar. While at the lower velocities with sufficient bath height freezing accounts for nearly all the coating, at the higher speeds and for shallower baths the time is not sufficient for freezing to reach an equilibrium value and the viscous process predominates. In fact, for any one bath height the total thickness of coating increases with fibre speed although the contribution made by freezing is reduced. The use of two different substrate radii is another test of the validity of the theories, since on the Deryagin theory the coating thickness should be proportional to the substrate radius, whereas on freeze-coating theory no obvious relation exists unless the coating is of maximum thickness, when it will also be proportional to the substrate radius.

The substrate ratio in these experiments was 1.20 and thickness ratios measured and calculated are given in Table 1.

Table 1. Ratio of coating thicknesses on thick and thin wires

Speed m/s	Freeze only (calculated)	Freeze + viscous (calculated)	Experimental mean of all the baths
0.254	1.02	1.05	1.07 ± 0.04
0.333	1.02	1.06	1.08 ± 0.03
0.419	1.02	1.07	1.09 ± 0.08
0.534	1.02	1.08	1.05 ± 0.06

For the 0.22 mm radius wire the Deryagin coating thickness for $\eta = 1.85 \times 10^{-3}$ and $\sigma = 2.68 \times 10^{-2}$, would be $4.98 \times 10^{-2} u^{2/3}$ mm. The actual thickness for the deep (24.5 mm) bath is 13.76×10^{-2} mm so that the radius R in equation (2) would need to be 0.94 mm for the Deryagin theory to hold. This "effective radius" for lower speeds in the deep bath has values 0.90, 0.96 and 1.12 mm. Hence if the Deryagin theory is correct in this region the coated wire needs to be already nearly 1 mm in radius, or about four times its final radius. Clearly the theory does not hold with the values of η and σ used.

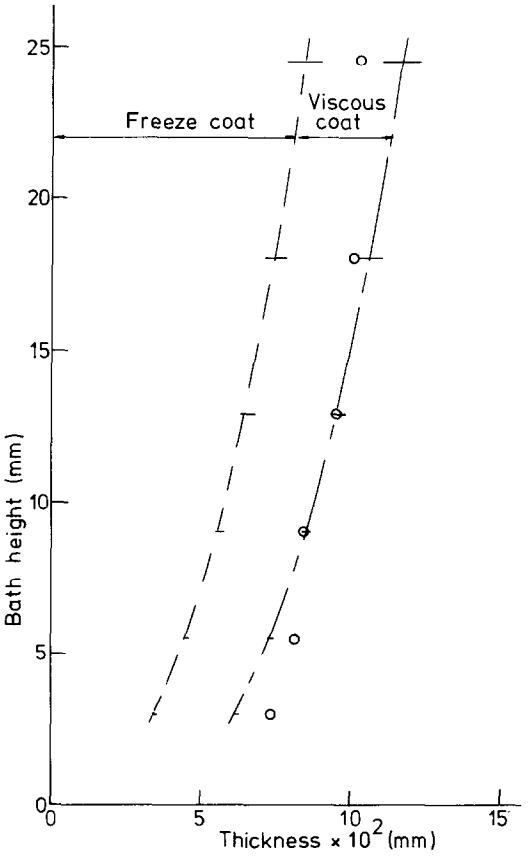

FIG. 5. As for Fig. 4 but velocity 0.333 m/s.

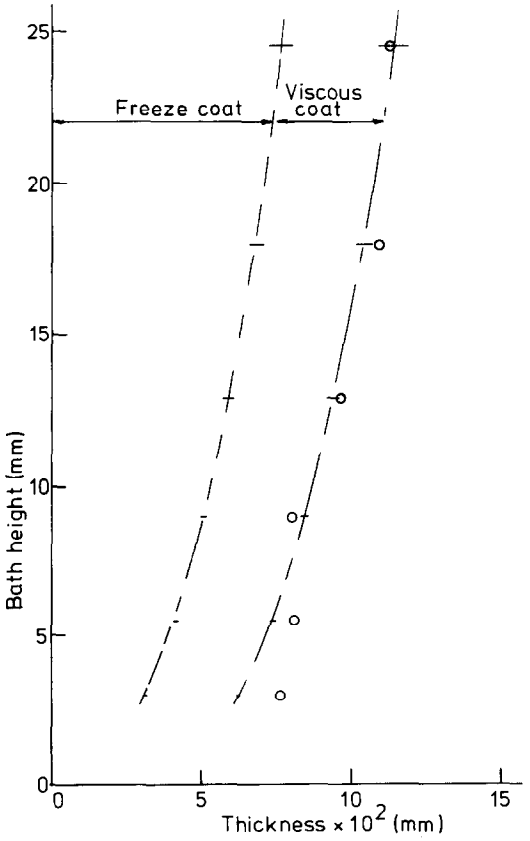

FIG. 6. As for Fig. 4 but velocity 0.419 m/s.

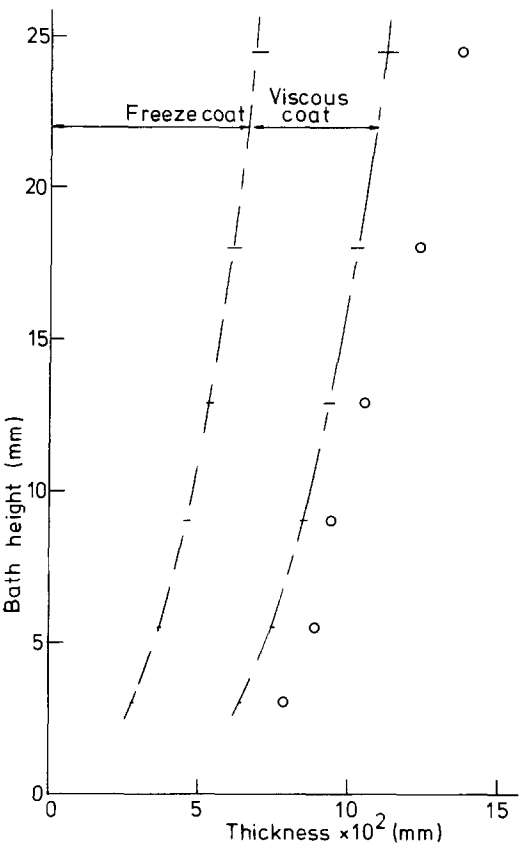

FIG. 7. As for Fig. 4 but velocity 0.536 m/s.

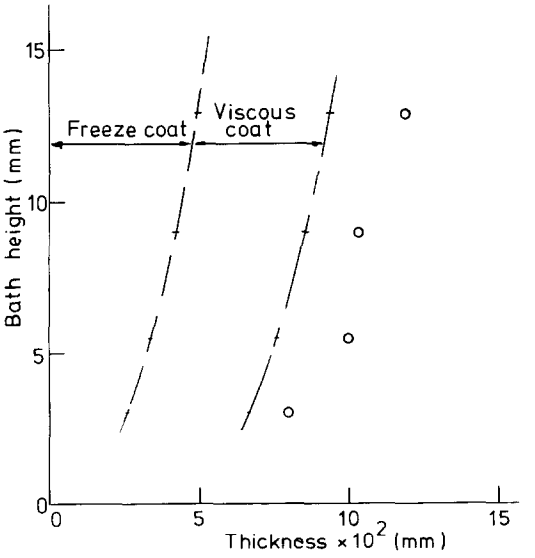

FIG. 8. As for Fig. 4 but velocity 0.646 m/s.

Now η may be much larger near freezing (the figures used were for η at 71°C and σ at 70) the bath temperature being 73°C and freezing point of biphenyl being 70°C. However η needs to be $\sim (4.3)^{3/2}$ or about nine times larger than its value at 71°C to give a correct Deryagin thickness. We still then need to account for the change with bath height, easily accounted for on our theory, but not on Deryagin's.

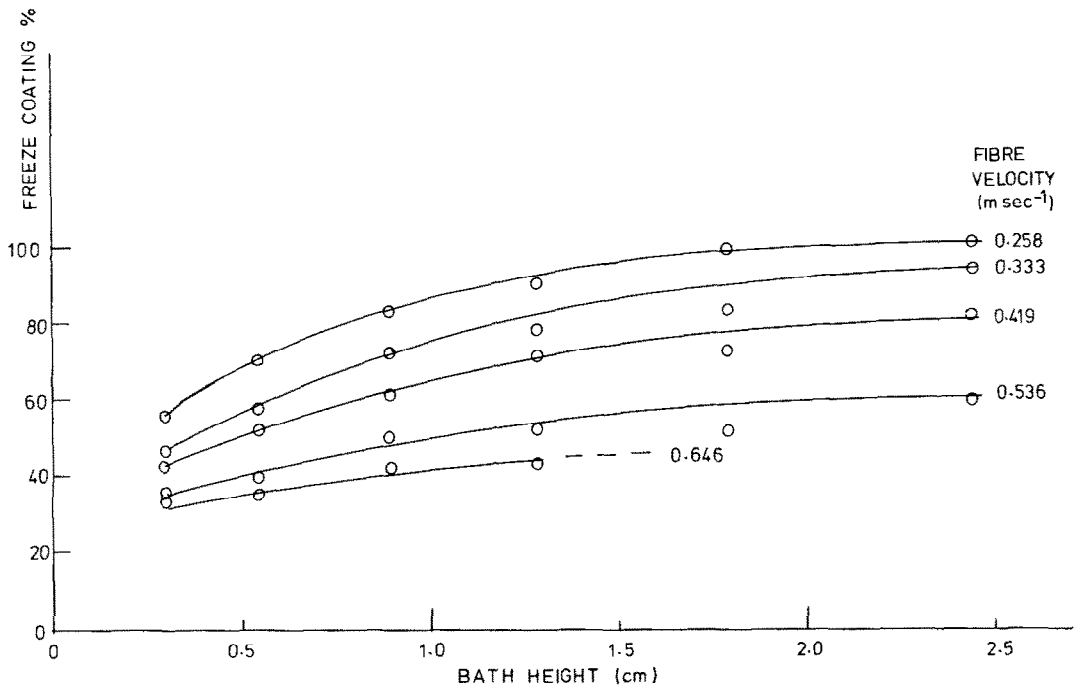

FIG. 9. The percentage of coating due to freezing as a function of bath height and fibre velocity.

The ratio: coating thickness/(velocity)$^{2/3}$ should be independent of velocity and bath height if the Deryagin theory is applicable. Table 2 contains values of this ratio which shows that it is not independent of velocity or bath height.

Table 2. The ratio coating thickness/(velocity)$^{2/3}$ $\times 10^4$

Wire radius (mm)	Bath depth (mm)	Velocity (m/s)				
		0.254	0.333	0.419	0.536	0.646
0.22	24.5	2.49	2.14	2.01	2.09	
	12.9	2.20	1.98	1.73	1.60	1.59
0.183	24.5	2.35	1.95	1.73	1.83	
	12.9	2.05	1.77	1.51	1.58	1.51

5. DISCUSSIONS AND CONCLUSIONS

The agreement of theory and experiment is good although there are directions in which refinement of the theory could be attempted. No account has been taken of convective effects, the model of freeze-coating being in essence a static one. Convective effects would tend to reduce the temperature gradient in the liquid or, to consider it another way, to bring the assumed outer boundary closer to the fibre. The net result would be to produce a thinner freeze-coat. Since these effects are likely to be more noticeable at high speeds the observations suggest that they are of small magnitude for the system studied. Turbulence at the speeds employed is not likely, Reynold's number for the highest speed being only 140.

Turning to the heat conduction equation, terms corresponding to axial flow along the fibre have not been included although heat absorbed at the fibre surface must diffuse radially inward and also axially outward both on the input side of the bath and at its upper surface.

The numerical calculations using radial symmetry show that for the present case of a poorly conducting layer laid on to a substrate fibre which is a good conductor the radial distribution of temperature rapidly becomes uniform. It is, therefore, a good approximation for the axial flow to consider flow along a thin rod heated at one end (Chapter 4 [9]). The solution, assuming no radiation loss is

$$v = (V_s - V_0)\,\mathrm{erfc}\,\frac{z}{2(\kappa t)^{1/2}}$$

where V_s is the surface temperature at $z = 0, t > 0$, V_0 is the initial temperature of the fibre and κ is the diffusivity.

The rate at which heat travels down the wire is small as compared with the fibre velocities used (minimum 0.25 m/s). Thus for the $v = 0.6\ (V_s - V_0)$ contour to reach $z = 0.79$ mm requires 10^{-2} s. But in this time the fibre travels 2.5 mm.

On the input side a steady temperature gradient will be established (in fixed axes), obtained by putting $t = z/u$ in the conduction equation.

Then

$$\frac{v}{V_s - V_0} = \mathrm{erfc}\,\frac{u}{2(\kappa)^{1/2}}\,z^{1/2}.$$

Typical values for $v/(V_s - V_0)$ at point z below the bath inlet are

$v/(V_s - V_0)$	z (m)
0.777	7.2×10^{-5}
0.479	4.51×10^{-4} (about one fibre diameter)
0.322	8.84×10^{-4}
0.203	1.46×10^{-3}.

Only small amounts of heat will therefore be absorbed axially, the amount can be considered as an end-correction of about 2 fibre diameters on the overall length in the bath.

The largest effect that we have not considered rigorously is that of the frozen-on coating on the calculated viscous layer. The steady state theory of Deryagin assumes a constant radius substrate upon which the viscous layer grows. In our case, however, the radius is increasing because of the freeze-coat so that, strictly, the Deryagin expression is not applicable.

The correct formula for the viscous layer growing on a substrate of increasing radius is likely to be complicated. We use the Deryagin expression assuming the final radius of the freeze-coat to be the substrate radius for the viscous coat. This should over-estimate the viscous coating.

We have not considered the effects of "restricted withdrawal" [3] to be relevant in this work since the bath/fibre diameter ratio was 15 or more and meniscus effects were unlikely. It is possible however that variations in the surface tension σ or the viscosity η with temperature in the neighbourhood of the freeze-coat could be responsible for some of the discrepancies between theory and experiment in this work.

In conclusion, this study of freeze- and viscous-coating of a thin wire, has shown that a reasonable prediction of the measured coating thickness can be obtained by addition to the freeze-coating thickness of a viscous layer calculated by Deryagin's theory. In this study, freeze-coating accounted for between 34% and 100% of the final coating thickness, depending upon the bath height and the fibre velocity.

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LE REVETEMENT DES FIBRES

Résumé—Une recherche expérimentale est faite sur le revêtement d'une fibre par un liquide dans lequel une part significative du revêtement est dû au gel. Des méthodes numériques sont données pour calculer l'épaisseur de la couche en résolvant le problème de Stefan avec introduction des termes de viscosité. Pour le système étudié (biphényl sur cuivre) le gel intervient pour 34–100% dans le revêtement, selon le temps d'immersion de la fibre dans le liquide et la vitesse de la fibre. Le rapport des épaisseurs des couches pour deux rayons différents du substrat est en bon accord avec les valeurs calculées. Les épaisseurs individuelles estimées diffèrent des valeurs observées de moins de 20%, l'accord étant moins bon pour les grandes vitesses des fibres.

DIE BESCHICHTUNG VON FASERN

Zusammenfassung—Es wird über eine experimentelle Untersuchung der Ummantelung einer Faser mit Flüssigkeit berichtet, wobei ein bedeutender Teil der Ummantelung durch Erstarren erfolgt. Numerische Methoden werden beschrieben, mit denen die Schichtdicke durch Lösung des entsprechenden Stefan-Problems und Addition von Zähflüssigkeitsbedingungen berechnet wird. Für das untersuchte System (Biphenyl auf Kupfer) wurden zwischen 34% und 100% der Schichtdicke durch Erstarren erlangt, abhängig von der Eintauchzeit der Faser in die Flüssigkeit und von der Geschwindigkeit. Das Verhältnis der Schichtdicken zu zwei verschiedenen Substrat-Durchmessern stimmte gut mit den vorhergesagten Werten überein. Die Voraussage der einzelnen Schichten wich bis zu 20% von den beobachteten Werten ab; für höhere Faser-Geschwindigkeiten war die Übereinstimmung schlechter.

НАНЕСЕНИЕ ПОКРЫТИЙ НА ВОЛОКНА

Аннотация — Проведено экспериментальное исследование нанесения покрытия на волокно в случае, когда значительная часть покрытия обусловлена намораживанием. Для расчёта толщины покрытия приведены численные методы решения соответствующей задачи Стефана с учётом вязкости. В исследуемой системе (бифенил на меди) намораживание обуславливает от 34% до 100% покрытия в зависимости от времени нахождения волокна в используемой для покрытия жидкости и скорости движения волокна. Полученные данные о толщине покрытия, выраженной через отношение радиусов внутренней и внешней поверхностей покрытия, хорошо согласуются с результатами расчетов. В отдельных случаях расхождение в результатах составляло 20%, причём большее отклонение соответствовало более высоким скоростям движения волокна.